V57/S24045/EE/20170728

Time : 3 Hours **Marks : 80**

Instructions :

1. All Questions are Compulsory.
2. Each Sub-question carry 5 marks.
3. Each Sub-question should be answered between 75 to 100 words. Write every questions answer on separate page.
4. Question paper of 80 Marks, it will be converted in to your programme structure marks.

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1. Solve any **four** sub-questions.

   a) The cost of maintenance of a machine is given as a function increasing with time and it's scrap value is constant. Then prove that machine should be replaced when the average annual cost to date becomes equal to the current maintenance cost. 5

   b) The annual demand for an item is 3200 units. The unit cost Rs. 6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150, determine: 5

      i) Economic order quantity

      ii) No. of order per year

      iii) Time between two consecutive orders

      iv) The optimal cost

   c) If in a particular single-server system, the arrival rate, $\lambda = 5$ per hour and service rate $\mu = 8$ per hour. Assume that conditions for use of the single channel queuing model, find out. 5

      i) The probability that the server is idle.

      ii) The probability that there are at least two customers in the system.

      iii) Expected time that a customer is in the queue.

      iv) Expected waiting time in queue plus service time.

      v) Average number of customers in the queue.

   d) Prove that $H(p_1, p_2, ...p_{n-1}, q_1, q_2, ..., q_m) = H(p_1, p_2, ..., p_n) + p_n H\left(\frac{q_1}{p_n}, \frac{q_2}{p_n}, ..., \frac{q_m}{p_n}, \right)$

   where the entropy function $H(p_1, p_2, ..., p_n)$ is given by $H(p_1, p_2, ..., p_n) = \sum_{i=1}^{n} p_i \log p_i$

   where $p_n = \sum_{k=1}^{m} q_k$. 5

e) A newspaper boy purchase paper for 25 paise each and sells them for 30 paise each. He can not return the unsold newspaper. Daily demand has the following distribution.

Demand	30	31	32	33	34	35	36
Probability	0.1	0.2	0.3	0.2	0.1	0.05	0.05

If each days demand is independent of the previous day, how many news paper should be order each days? 5

2. Solve any **four** sub-questions.

a) Draw the network diagram and determine the critical path for following project. 5

Activity	Time estimate (weeks)	Activity	Time estimate (weeks)
1-2	5	3-7	7
1-3	6	4-7	4
1-4	3	5-8	2
2-5	5	6-8	5
3-6	7	7-8	6
		8-9	4

b) A hospital clinic has a doctor examining patients brought in for a general check up the doctor average 4 minutes on each phase of the check up although the distribution of time spent on each phase is approximately exponential. If each patient goes though through four phase in the check up and if the arrivals of the patient in the doctors office are approximately Poisson at an average rate of 3 per hour, 5

i) What is the average time spend by patient waiting in the doctors office?

ii) What is the average time spent in the examination?

iii) What is the most probable time spent in the examination?

c) The probability distribution of monthly sales of a certain item is follows: 5

Monthly sate in units	0	1	2	3	4	5	6
Probability	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs. 10/- per unit per month. The current policy is to maintain a stock of four item at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one item for one unit of time.

d) At the time zero, all items in a system are new. Each item has probability p of failing immediately before the end of first month of life and probability $q = 1 - p$ of failing immediately before the end of second month. If all the items are replaced as they fail. Show that the expected number of failure $f(x)$ at the end of the month x is given by 5

$$f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$$

Where N is the number of items in the system.

e) In the case of 'Queue with discouragement' prove that P_n follows the Poisson distribution. 5

3. Solve any **four** sub-questions.

a) Find the present value of the total expenditure on the machines with replacement policy of *n* years of items whose maintenance cost increase with time and the value of money also change with time. 5

b) A manufacturer has no supply his customer 600 units of his product per year. Shortages are not allowed and the storage cost amount to Rs. 0.60 per unit per year. The set up cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost. 5

c) If at any time *t*, there are *n* consumers in the queue for the supply of current and $\lambda_n = (a-n)\lambda, \mu_n = n\,\mu, 0 \le n \le a$ then prove that $P_n = \binom{a}{n}\left(\frac{\lambda}{\mu+\lambda}\right)^n \left(\frac{\mu}{\mu+\lambda}\right)^{a-n}$ 5

d) Show that the entropy of the following probability distribution is $2-\left(\frac{1}{2}\right)^{n-2}$ 5

Event	x_1	y_2	-	-	x_i	-	-	x_{n-1}	x_n
Probabilities	$\frac{1}{2}$	$\frac{1}{2^2}$	-	-	$\frac{1}{2^i}$	-	-	$\frac{1}{2^{n-1}}$	$\frac{1}{2^n}$

e) The activities involved in a PERT project are given in table. 5

Job	Duration (days)		
i-j	t_o	t_m	t_p
1-2	3	6	15
2-3	6	12	30
3-5	5	11	17
7-8	4	19	28
5-8	1	4	7
6-7	3	9	27
4-5	3	6	15
1-6	2	5	14
2-4	2	5	8

i) Find expected task time and their variance.
ii) Draw a network diagram.
iii) Find the critical path after estimating the earliest and latest event time for all nodes.
iv) Find the probability of completing the project before 31 days.

4. Solve any **four** sub-questions.

a) An airline requires 200 assistant hostesses, 300 hostesses and supervisors girls are required all the age 21. If still in service the retire at the age of 60. Given the following life table determine : 5

i) How many girls should be recruited each year?

ii) At what ages promotion should take place?

b) Explain time estimate in critical path analysis for 5

i) Forward pass method

ii) Backward pass method

c) Problem arrives at a computing centre in a passion fashion at an average rate of five per day. The values of the computing centre are that any man waiting to get his problem solved must aid the man whose problem is being solved. It the time to solved problem with one man has an exponential distribution with mean time of $\frac{1}{3}$ day and if the average solving time is inversely proportional to the number of people working on the approximate the expected time in the centre of a person entering the line. 5

d) Derive an economic lot size formula assumption 5

i) λ = the demand for product in one unit of time.

ii) Production rate is infinite

iii) Lead time is zero

iv) P = Price of one item of product in Rupees

v) I = cost of carrying one Rupee to the inventory for one year

vi) C_3 = set up cost per day.

vii) Shortage are not allowed.

e) A firm is considering replacement of a machine, whose cost price is Rs. 1,20,000 and the scrap value is Rs. 20,000. The running (maintenance and operating) cost of the machine are as follows: 5

Year	1	2	3	4	5	6	7	8
Running cost (Rs.)	2000	5000	8000	12,000	18,000	25,000	32,000	40,000

When should be machine replaced?

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