V57/S24013/EE/20160712

Time : 3 Hours **Marks : 80**

Instructions :

1. All Questions are Compulsory.
2. Each Sub-question carry 5 marks.
3. Each Sub-question should be answered between 75 to 100 words. Write every questions answer on separate page.
4. Question paper of 80 Marks, it will be converted in to your programme structure marks.

---

1. Solve any **four** sub-questions.

   a) Let $\{E_k\}_{k=1}^{\infty}$ be countable collection of measurable sets of real numbers then show that $m^*\left(\bigcup_{k=1}^{\infty} E_k\right) \leq \sum_{k=1}^{\infty} m^*(E_k)$ 5

   b) If E_1 is measurable set and $m*(E_1 \Delta E_2) = 0$ then show that E_2 is also measurable. 5

   c) If $E_1, E_2, ..., E_N$ is a finite collection of measurable sets then show that $\bigcup_{k=1}^{N} E_k^c$ is again measurable. 5

   d) State and prove Nested set Theorem. 5

   e) State and prove Monotonicity property of outer measure. 5

2. Solve any **four** sub-questions.

   a) Let $f : D \rightarrow IR$ be an extended real valued function such that D is measurable set, then show that the set $\{x/ f(x) > c\}$ is measurable $\Leftrightarrow \{x/ f(x) \geq c\}$ is measurable $\forall c \varepsilon IR$. 5

   b) Let $f(x)$ and $g(x)$ be measurable functions defined on E that is finite a.e. on E, then show that for any α and β, $\alpha f(x) + \beta g(x)$ is also measurable on E. 5

   c) Give an example to show that if $\{f_n(x)\}$ is sequence of Riemann integrable functions which converges to $f(x)$ but still $f(x)$ is not Riemann integrable. 5

d) Show that if A = [0,1] and B = [2,3] then $\int_{A\cup B} f(x)dx = \int_A f(x)dx + \int_B f(x)dx$, where $f(x)$ is bounded measurable function defined on $A \cup B$. 5

e) Let $\{f_n(x)\}$ be a sequence of bounded measurable functions on a set of finite measure and if $f_n(x) \to f(x)$ on E uniformly show that $\lim_{n\to\infty} \int_E f_n(x)dx = \int_E f(x)dx$ 5

3. Solve any **four** sub-questions.

a) State and prove Monotone Convergence Theorem. 5

b) If $f(x) = \sin x,\ x \in [0, 2\pi]$ then find $f^+(x)$ and $f^-(x)$ of $f(x)$. 5

c) State and prove Lebesgue convergence Theorem. 5

d) State and prove Jordan's Theorem. 5

e) Show that every increasing function on [a,b] is of bounded variation. 5

4. Solve any **four** sub-questions.

a) If $f(x)$ satisfies Lipchit'z condition on closed and bounded interval [a,b] then show that $f(x)$ is absolutely continuous on it. 5

b) State and prove fundamental Theorem of integral Calculus for Lebesgue integral. 5

c) Let $f(x)$ be integrable over [a,b] then show that $f(x) = 0$ for almost $\forall x \in [a,b] \Leftrightarrow \int_c^d f(x)dx = 0, \forall (c,d) \subseteq [a,b]$. 5

d) State and prove Young's Inequality. 5

e) State and prove Cauchy-Schwartz inequality. 5

✦✦✦✦✦✦✦