V57/S24011/EE/20160708

Time : 3 Hours **Marks : 80**

Instructions :

1. All Questions are Compulsory.
2. Each Sub-question carry 5 marks.
3. Each Sub-question should be answered between 75 to 100 words. Write every questions answer on separate page.
4. Question paper of 80 Marks, it will be converted in to your programme structure marks.

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1. Solve any **four** sub-questions.

a) H is a subgroup of group G and N is a normal subgroup of G Then prove that $\frac{HN}{N} \cong \frac{H}{H \cap N}$ 5

b) Show that if $\{e\} = H_0 < H_1 < H_2 < ..... < H_n = G$ is a subnormal series of a group G and $o\left(\frac{H_{i+1}}{H_i}\right) = S_{i+1}$ then G is of finite order $S_1 * S_2 * S_3 * .... * S_n$. 5

c) Find the ascending central series for : 5

i) S_3

ii) D_4

d) Prove that : Any group of order p^n is nilpotent. 5

e) Let G be a group and let G' be the derived subgroup of G. Then prove the following 5

i) $G' \triangleleft G$ i.e. G' is normal subgroup of G.

ii) $\frac{G}{G'}$ is abelian.

2. Solve any **four** sub-questions.

a) State and prove Second Sylow theorem. 5

b) Let *G* be a finite group with $|G| = p * q$ where *p* and *q* are distinct primes and *p*<*q*, then prove the following 5

i) *G* contains a normal subgroup of order *q*.

ii) *G* is not simple

c) Show that a group of order 108 can not simple. 5

d) If $o(G) = p^2$ (p is prime), then prove that G is an abelian group. 5

e) Find class equation of S_4. 5

3. Solve any **four** sub-questions.

a) Prove that R is an integral domain iff $R[x]$ is an integral domain. 5

b) Let D be a UFD. $f(x), g(x) \in D[x]$ be primitive polynomials. Then $f(x) * g(x)$ is also primitive in $D[x]$. 5

c) Let F be a field then prove that $< p(x) > \neq \{0\}$ of $F(x)$ is maximal iff $p(x)$ is irreducible over F. 5

d) Show that $f(x) = 8x^3 - 6x - 1 \in Z[x]$ is irreducible over Q. 5

e) Show that the polynomial $(x^4 + 4)$ can be factored into linear factors in $Z_5[x]$. 5

4. Solve any **four** sub-questions.

a) Prove : For any module homomorphism $f : M \to N$, ker f is a submodule of the module M and im f is a submodule of the module N. 5

b) Let A and B be R-submodules of an R-module M. Then prove that: $\frac{A+B}{A} \cong \frac{B}{A \cap B}$ 5

c) Let M be R-module and N be R-submodule of M. Then prove that M is Noetherian iff N and M/N are Noetherian. 5

d) Let V=R^3 be a vector space over the field R. Let $x_1 = (1,0,0)$, $x_2 = (1,1,0)$, $x_3 = (1,1,1)$ show that $V = Rx_1 + Rx_2 + Rx_3$. 5

e) Show that: Let M be a R-module. let $K \subset N \subset M$ are submodules of M. If K is a direct summand of M and if N/K is a direct summand of M/K then prove that N is a direct summand of M. 5