
Instruction :

1. All Questions are Compulsory.
 2. Each Sub-question carry 5 marks.
 3. Each Sub-question should be answered between 75 to 100 words. Write every questions answer on separate page.
 4. Question paper of 80 Marks, it will be converted in to your programme structure marks.
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1. Solve any **four** sub-questions.
 - a) Outline the key differences between deterministic models and stochastic models. Give an example each of deterministic and stochastic models. 5
 - b) A new town is planned in a currently rural area. A model is to be developed to recommend the number and size of schools required in the new town. The proposed modelling approach is as follows: 5
 - The current age distribution of the population in the area is multiplied by the planned population of the new town to produce an initial population distribution.
 - Current national fertility and mortality rates by age are used to estimate births and deaths.
 - The births and deaths are applied to the initial population distribution to generate a projected distribution of the town's population by age for each future year, and hence the number of school age children.Discuss the appropriateness of the proposed modelling approach.
 - c)
 - i) Explain how the classification of stochastic processes according to the nature of their state space and time space leads to a four way classification. 1
 - ii) For each of the four types of process give an example of a statistical model. 4
 - d) Define each of the following:
 - i) White noise 1
 - ii) Poisson process 2
 - iii) Filtration 2
 - e) X_t is a simple random walk with p as the increment probability. 5
Calculate $P(X_{20} = 20|X_0 = 0)$, $P(X_{10} = 0|X_0 = 0)$, $P(X_{20} = 5|X_0 = 0)$, $P(X_{100} = 10|X_0 = 0)$.

2. Solve any **four** sub-questions.

- a) In a game of tennis, when the score is at “Deuce” the player winning the next point holds “Advantage”. If a player holding “Advantage” wins the following point that player wins the game, but if that point is won by the other player the score returns to “Deuce”.

When Andrew plays tennis against Ben, the probability of Andrew winning any point is 0.6. Consider a particular game when the score is at “Deuce”.

- i) Show that the subsequent score in the game can be modelled as a Markov Chain, specifying both: 3

p) The state space; and

q) The transition matrix

- ii) State, with reasons, whether the chain is: 2

p) Irreducible; and

q) Aperiodic

- b) A.R. Rehman a musician has to compose Eight tracks for a movie and typically he works on one track each week. Every Friday, A.R. Rehman critically reviews the track he has just composed. There is an 80% chance that he will be happy with his work and the track will be treated as final. However, there is a 20% chance that Mr. Rehman will not be satisfied; and will dismiss the just-composed track. Once A. R. Rehman has finalized a track, he does not revisit it at any later date.

Let T_k denote the number of tracks that Mr. Rehman has completed by the end of the k^{th} week.

Let $T_0 = 0$

- i) Explain why T_k can be modelled as a Markov Chain. 2

- ii) Calculate the probability that A.R. Rehman will compose all 8 tracks in: 3

p) Exactly 8 weeks

q) Exactly 12 weeks

- c) A motor insurer’s No claims discount system uses the following levels of discount {0%, 25%, 40%, 50% }. Following a claim free year a policyholder moves up one discount level (or remains on 50% discount). If the policyholder makes one (or more) claims in a year they move down one level (or remain at 0% discount).

The insurer estimates that the probability of making at least one claim in a year is 0.1 if the policyholder made no claims the previous year, and 0.25 if they made a claim the previous year.

New policyholders should be ignored.

- i) Explain why the system with state space {0%, 25%, 40%, 50% } does not form a Markov Chain. 2

- ii) p) Show how a Markov Chain can be constructed by the introduction of additional states.

q) Write down the transition matrix for this expanded system, or draw its transition diagram.

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- d) Employees of a company are given a performance appraisal each year. The appraisal results in each employee's performance being rated as High (H), Medium (M) or Low (L). From evidence using previous data it is believed that the performance rating of an employee evolves as a Markov Chain with transition matrix.

$$P = \begin{matrix} & \begin{matrix} H & M & L \end{matrix} \\ \begin{matrix} H \\ M \\ L \end{matrix} & \begin{pmatrix} 1-\alpha-\alpha^2 & \alpha & \alpha^2 \\ \alpha & 1-2\alpha & \alpha \\ \alpha^2 & \alpha & 1-\alpha-\alpha^2 \end{pmatrix} \end{matrix}$$

for some parameter α .

- i) Determine the range of values for α for which the matrix P is a valid transition matrix. 2
- ii) Explain whether the chain is irreducible and / or aperiodic. 3
- e) A no claims discount system operates with four levels of discount 0%, 20%, 40% and 50%.

If a policyholder makes no claim during the year he moves up a level of discount (or remains at the maximum level). If he makes one claim during the year he moves down one level of discount (or remains at the minimum level) and if he makes two or more claims he moves down to, or remains at, the minimum level.

The probability for a policyholder making two or more claims in a year is 25%.

The long-term probability of being at the 0% discount level is the same as the long-term probability of being at the 50% discount level.

Derive the probability of a policyholder making exactly one claim in a given year.

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3. Solve any **four** sub-questions.

- a) Observation of life i begins at exact age 82 years and 3 months. Observation will continue until the earlier of the life's 83rd birthday or death. 2
- i) State the value or range of values taken by:
- p) a_i, b_i
- q) V_i
- ii) Let the transition intensity μ equal 0.1. Find
- p) The probability function of D_i 1
- q) The probability density/mass function of V_i 2

- b) A scientist identifies 1,100 newborn kittens and observes them during their first year of life. Scientist wishes to calculate a constant force of mortality (μ) in the first year covering all causes of death. If the true μ is 0.036, calculate the probability that the hazard rate observed by the scientist is greater than 0.04. 5
- c) An investigation was carried out into the relationship between sickness and mortality in an historical population of working class men. The investigation used a three-state model with the states:
- Healthy
 - Sick
 - Dead

Let the probability that a person in state i at time x will be in states j at time $x + t$ be ${}_x p_x^{ij}$. Let the transition intensity at time $x + t$ between any two state i and j be μ_{x+t}^{ij} . The investigation collected the following data:

- Man -years in Healthy state 265
 - Man-years in sick state 140
 - Number of transitions from Healthy to sick 20
 - Number of transitions from Sick to Dead 40
- Derive the maximum likelihood estimator of the transition rate from sick to dead. 3
 - Hence estimate the value of the constant transition rate from sick to dead. 2
- d) A government has introduced a two-tier driving test system. Once someone applies for a provisional licence they are considered a Learner driver. Learner drivers who score 90% or more on the primary examination (which can be taken at any time) become qualified. Those who score between 50% and 90% are obliged to sit a secondary examination and are given driving status restricted. Those who score 50% or below on the primary examination remain as Learners. Restricted drivers who pass the secondary examination become qualified, but those who fail revert back to learner status and are obliged to start again.
- Sketch a diagram showing the possible transitions between the states. 2
 - Write down the likelihood of the data, assuming transition rates between states are constant over time, clearly defining all terms you use. 3

- e) A Markov process has two states, labelled state A and state B. You are given that $\mu_{AB} = 0.02$ and $\mu_{BA} = 0.08$.
Write down Kolmogorov's forward differential equation for the probability $p_{AB}(t)$ and solve this differential equation to obtain an expression for $p_{AB}(t)$. 5

4. Solve any **four** sub-questions.

- a) A continuous time Markov process with states { Able to work (A), Temporarily unable to work (T), Permanently unable to work (P), Dead (D) } is used to model the cost of providing an incapacity benefit when a person is permanently unable to work. The generator matrix, with rates expressed per annum, for the process is estimated as:

	A	T	P	D
A	-0.15	0.1	0.02	0.03
T	0.45	-0.6	0.1	0.05
P	0	0	-0.2	0.2
D	0	0	0	0

- i) Calculate the probability of a person remaining in state A for at least 5 years continuously. 2
- ii) Define $F(i)$ to be the probability that a person, currently in state i , will never be in state P. 3
Derive an expression for:
p) $F(A)$ by conditioning on the first move out of state A.
q) $F(T)$ by conditioning on the first move out of state T.
- b) For Poisson process $N(t)$ with parameter λ , derive an expression for difference differential equation for $P_n(t) = P(N(t) = n)$ from first principle. Hence evaluate $p_0(t)$. 5
- c) Marital status is considered using the following time-homogeneous, continuous time Markov jump process:
- The transition rate from unmarried to married is 0.1 per annum.
 - The divorce rate is equivalent to a transition rate of 0.05 per annum.
 - The mortality rate for any individual is equivalent to a transition rate of 0.025 per annum, independent of marital status.

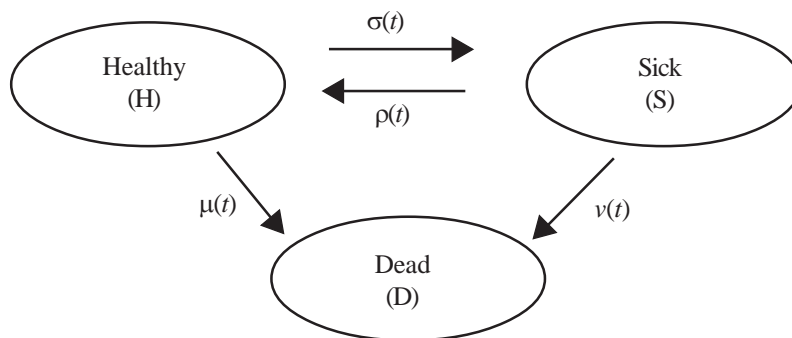
The state space of the process consists of five states: Never Married (NM), Married (M), Widowed (W), Divorced (DIV) and Dead (D).

P_x is the probability that a person currently in state x , and who has never previously been widowed, will die without ever being widowed.

- i) Construct a transition diagram between the five states. 2
- ii) Calculate the probability of never being widowed if currently in state NM. 2
- iii) Suggest two ways in which the model could be made more realistic. 1

- d) The occurrence of hurricanes in a county can be modeled as a Poisson process. Twenty hurricanes have touched down in a county within the last twenty years. If there is at least one hurricane occurring in a year, that year is classified as a 'hurricane year'.
- What is the probability that next year will be a 'hurricane year'? 2
 - What is the probability that there will be two 'hurricane years' within the next three years? 3
- e) i) p) Explain what is meant by a Markov jump process. 2
- q) Explain the condition needed for such a process to be time-homogeneous. 2
- ii) Outline the principal difficulties in fitting a Markov jump process model with time-inhomogeneous rates. 2

A company provides sick pay for a maximum period of six months to its employees who are unable to work. The following three-state, time-inhomogeneous Markov jump process has been chosen to model future sick pay costs for an individual:



Where sick means unable to work and Healthy means fit to work.

The time dependence of the transition rates is to reflect increased mortality and morbidity rates as an employee gets older. Time is expressed in years.

- iii) Write down Kolmogorov's forward equations in matrix form for this process, specifying the appropriate transition matrix. 1

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